

Household Analysis Review Group (HARG)

Review of the Household Projections Methodology

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The results presented in this paper are provisional.

They will differ from the final household projections published.

Please treat these figures as 'restricted' until after publication next year,
and do not pass them on to anyone else without prior agreement from National Records of
Scotland (NRS).

1. Purpose

In this paper we will compare various methods of producing household projections following the release of 2011 Census data. The aim of this is to help to select the most suitable method of projection, not only for the next set of household projections but also for subsequent publications.

2. Background

2.1 Comparisons between household estimates and projections, and the 2011 Census results

We published a [Reconciliation Report](#) on our website comparing the results of our household estimates and projections to the 2011 Census results, in June.

2.2 Current Method

The household projections are based on the sub-national population projections that are also produced by the National Records of Scotland (NRS).

A range of administrative data sources are used to calculate the number of people living in communal establishments, which is subtracted from the total population to estimate the number of people living in private households.

Information on the types of households, in the form of ‘headship rates’, is taken from 1991 and 2001 Censuses. The headship rates are the proportion of individuals of a particular age and gender within a local authority area who head households, where the ‘head’ of the household is normally the first adult entered on a census form. The non-headship rate is the proportion of people who are not heads of households within each age group, gender and local authority area.

These headship and non-headship rates from the 1991 and 2001 Censuses are fitted to a two point exponential curve and projected forward across the 25 year projection period. Applying these rates to the projected private household population gives the number of households by type of household, age of the head of household and local authority. Local authority figures are then constrained to the Scottish household projections and the two most recent household estimates and adjusted to ensure that there are enough people in the population projections to fill all the households.

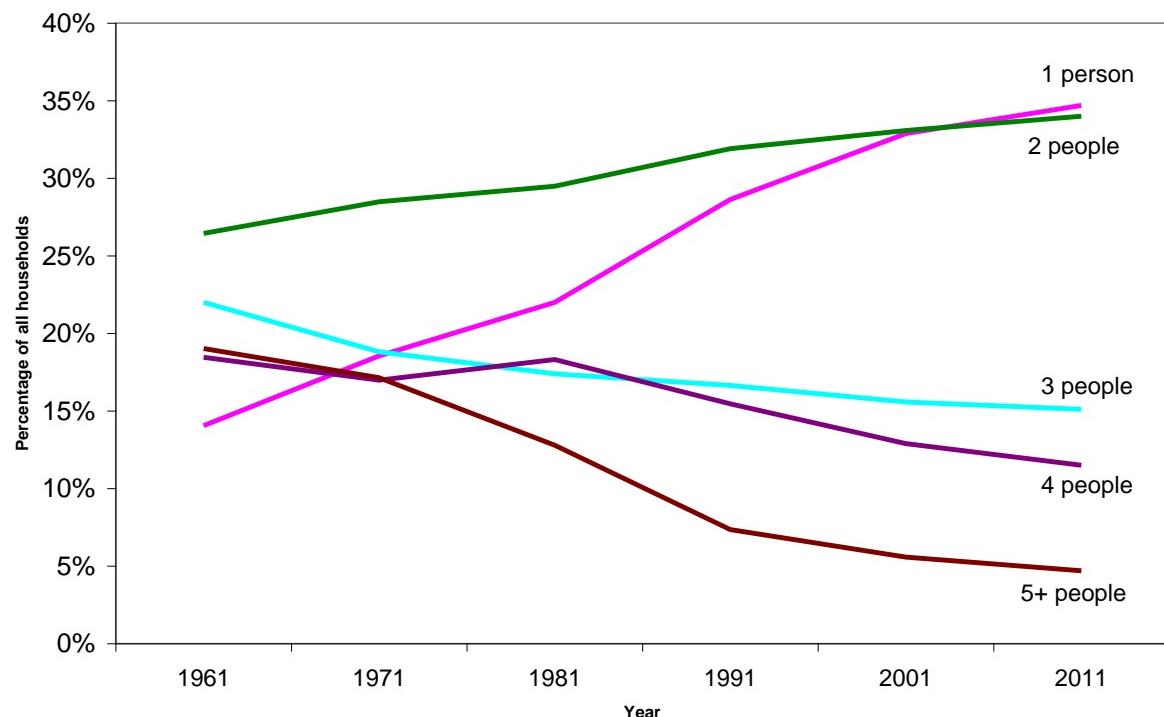
2.3 Why change the method?

If we continue to produce household projections with the current method (but updating the figures to use the 2001 and 2011 Census results instead of 1991 and 2001), we will be projecting headship rates based entirely on the trends from 2001 to 2011. This means that if the trends change then it can lead to the projections over-estimating certain household types and under-estimating others. [Figure 1](#) shows how the types of households people live in has changed in the last 50 years from the 1961 Census to the 2011 Census. As an example, in [Figure 1](#), for one person households the rate of increase is noticeably less between 2001 and 2011 than between 1991 and 2001. This change in the trend meant that the principal projection in the 2010-based projections over-estimated the number of one person households when compared to the 2011 Census, although the alternative headship variant, which we produced to show what would happen if there were slower rates of household change, was closer to the 2011 Census results, as shown in [Figure 2](#).

Using three censuses rather than two censuses would mean that the trends used in the projection would come from a longer period, which could help to reduce the impact of future trends differing from those observed between the 2001 and 2011 Censuses.

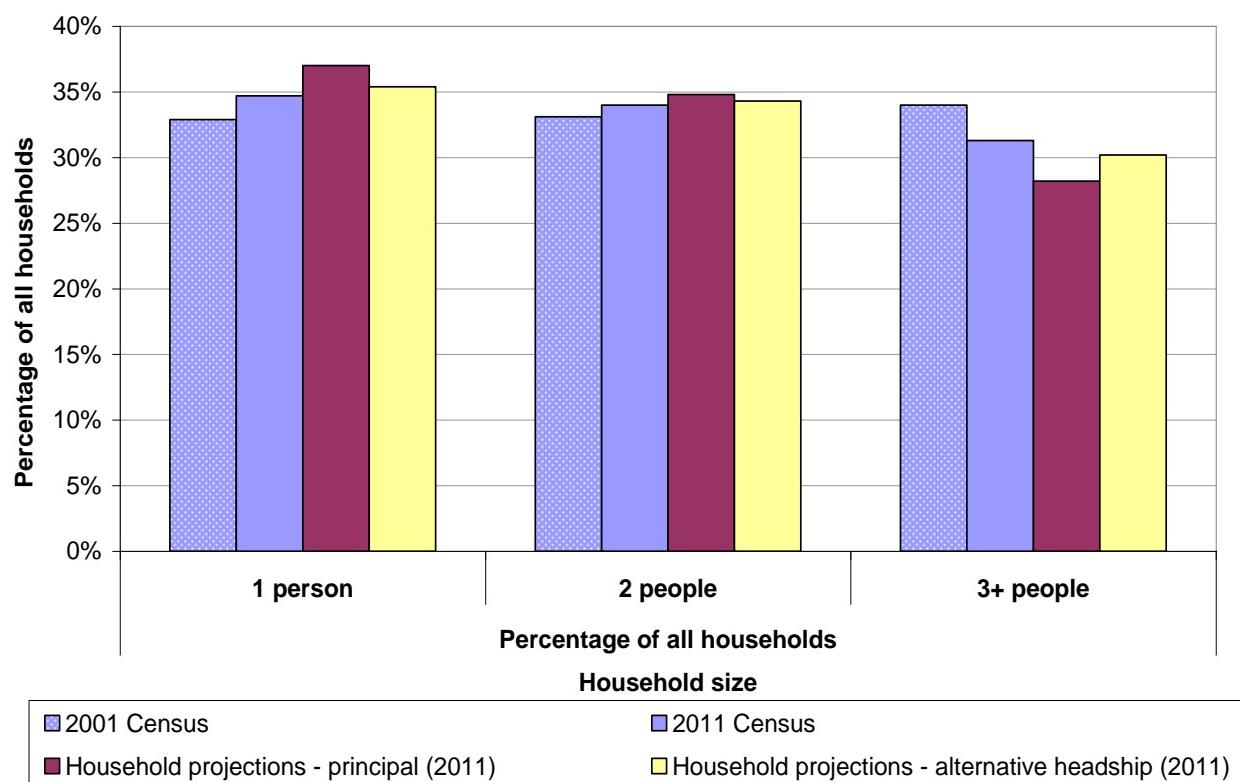
It would also be beneficial to have a method that provides some flexibility so that we can adjust the projections when data shows a change in household trends, rather than waiting for new census data. With the current method we are limited to producing variants, such as the alternative headship variant in the 2010-based projections where the headship rates changed at half the rate given in the principal projections.

Figure 1: Household size in Scotland, 1961-2011



Source: Census data

Figure 2: Comparison of different estimates of the distribution of household sizes in Scotland



3. Testing

In this paper, we test several different ways of producing household projections, and provide a summary of the results.

3.1 Methods tested

As the next set of household projections will include data from the 2011 Census, we wanted to consider how many censuses we should use when projecting headship rates. If we want to use more than two censuses this will also involve changing the method of projection as the current method only works with two censuses.

Two-point exponential method – This is the current method as described previously. This method only projects the headship rates from two censuses. The formula below projects the rates from the 2001 and 2011 Censuses:

There are four variations of this method:

- Using the 1991 and 2001 Censuses. This is what we would project if we continued with the current method and there was no new census data available. While this method would not actually be used to produce new sets of projections, it has been included to give an indication of how the projections will change.
- Using the 2001 and 2011 Censuses. This option continues the use of the current method, but uses the most recent censuses. With this method it would also be possible to use the Household Reference Person (HRP) rather than head of household as in the current method. The HRP is chosen from a household on the basis of their economic activity (in the priority order: full-time

job, part-time job, unemployed, retired, other). If more than one person has the same economic activity, the HRP is identified as the oldest of these people or, if they are the same age, the first of these people entered on the census form. This is the only method where we tested the HRP as it is only available to a 10 per cent sample of the 1991 Census.

- Using the 1991 and 2011 Censuses. Using these censuses projects the trend over 20 years rather than a 10 year trend like the other versions of this method. This would strike a balance between household trends that occurred before and during the global economic downturn.

Three-point exponential method – This is an extension of the current method that projects headship rates from all three censuses rather than just two. Annex A contains details on the mathematics behind this method of projection.

As with the two-point method using the 1991 and 2011 Censuses, the advantage of this method is that it looks at trends over a longer period of time than the current method, reducing the impact of events such as the global economic downturn or the EU accession in 2004. Unlike the two-point method using 1991 and 2011 Censuses, this method does not ignore the results of the 2001 Census.

Weighted two-point exponential method – This method creates several sets of projected headship rates by using the two-point exponential method using different pairs of censuses. The projected headships are then given weights and combined to produce a single set of projected headship rates.

For example, suppose we have two sets of projected headship rates, $y_i^{(1)}$ and $y_i^{(2)}$. We then combine these to produce a final set of projected headship rates, y_i , by choosing weights $A^{(1)}$ and $A^{(2)}$ and using the following formula:

$$y_i = \frac{A^{(1)} * y_i^{(1)} + A^{(2)} * y_i^{(2)}}{A^{(1)} + A^{(2)}}$$

The results for this method shown in this paper come from combining headship rates projected from the 1991 and 2001 Censuses with headship rates projected from the 2001 and 2011 Censuses. The weights were chosen to minimise the difference between the projected distribution of household types and distribution of household types in the 2012 Scottish Household Survey (SHS) data. The reason for this is that in the final projections, the total number of households would be adjusted using the household estimates while there are no adjustments made to household types. The final weightings were 74 per cent towards the headship rates projected from the 2001 and 2011 Censuses and 26 per cent towards the headship rates projected from the 1991 and 2001 Censuses.

We also considered combining headship rates projected from the 1991 and 2011 Censuses with rates projected from the 2001 and 2011 Censuses. However when attempting to minimise the difference in the projected distribution of household types from the SHS data it was found that the weighting would be 100 per cent towards the headship rates projected from the 1991 and 2011 Censuses, meaning that it would just be the two-point exponential method using 1991 and 2011 Censuses. This weighting was also found to minimise the difference between the projected number of households and the household estimates.

Again an advantage of this method is that it looks at trends over a longer period of time than the current method. A further advantage that this method has over the other methods is that the choice of weights would provide some flexibility to adjust the projections to trends observed from more recent data, such as from the Scottish Household Survey.

3.2 Data used in this paper

Population projections – The 2010-based population projection was used, as these are the most recent figures available. However this was adjusted to the 2011 and 2012 population estimates so that data from the 2011 Census was incorporated. When the 2012-based sub-national population projections are published next year, we will be able to use these figures. However, in order to allow us to publish the household projections as soon as possible after the population projections, we want to do this analysis with the data that is available now.

Communal establishment rates – The communal establishment rates used were an estimate of the population living in communal establishments in 2011 as 2012 data was not available. They are based on figures collected by our branch (they are not from the 2011 Census). While most of the data was from 2011, some was from 2009-10 and a small amount from the 2001 Census.

Headship rates – Headship rates derived from the 1991, 2001 and 2011 Censuses were used in the testing, with some methods using headship rates from two censuses and others using headship rates from all three censuses.

Household estimates – The 2012 household estimates were used to check how closely the projections agreed in the total number of households.

2012 Scottish Household Survey Data – This data was used to judge how the methods projected each household type. It was also used when selecting weights for the weight two-point exponential method as the weights were chosen to minimise the differences between the projection and the SHS data.

4. Initial Results

The results in this section were produced when the methods were used without constraining the projections to the household estimates and without minimum adult/child adjustments being performed. The reason for doing this was so that the projections could be compared to the household estimates as it would be preferable to choose a method that requires a minimal amount of adjustments.

The published 2010-based figures are an exception to this and have been included to highlight the effect of constraining to the household estimates, as without this constraining the 2010-based figures would be very similar to the two-point exponential method using the 1991 and 2001 Censuses.

The projections in the tables and charts are as follows:

Published 2010-based: These are the final figures from the most recent publication of household projections, so have been constrained to the household estimates.

Two-point using 1991 and 2001: Projections produced using the two-point exponential method, using headship rates based on the head of household from the 1991 and 2001. This is similar to what the published 2010-based projections were before constraining, although the input data has been updated.

Two-point using 1991 and 2011: Projections produced using the two-point exponential method, using headship rates based on the head of household from the 1991 and 2011 Censuses.

Two-point using 2001 and 2011: Projections produced using the two-point exponential method, using headship rates based on the head of household from the 2001 and 2011 Censuses. This is the method that would be used if we updated the previous methodology with the latest figures, with no changes to the methodology itself.

Two-point using HRP: Projections produced using the two-point exponential method, using headship rates based on the [household reference person](#) from the 2001 and 2011 Censuses.

Three-point: Projections produced using the three-point exponential method, using headship rates based on the head of household from the 1991, 2001 and 2011 Censuses.

Weighted two-point: Projections produced using the [weighted two-point method](#), combining headship rates projected from the 1991 and 2001 Censuses with headship rates projected from the 2001 and 2011 Censuses. Headship rates were based on the head of household. This projection had a weighting of 74 per cent towards the projection produced using the 2001 and 2011 Censuses and a weighting of 26 per cent towards the projection produced using the 1991 and 2001 Censuses.

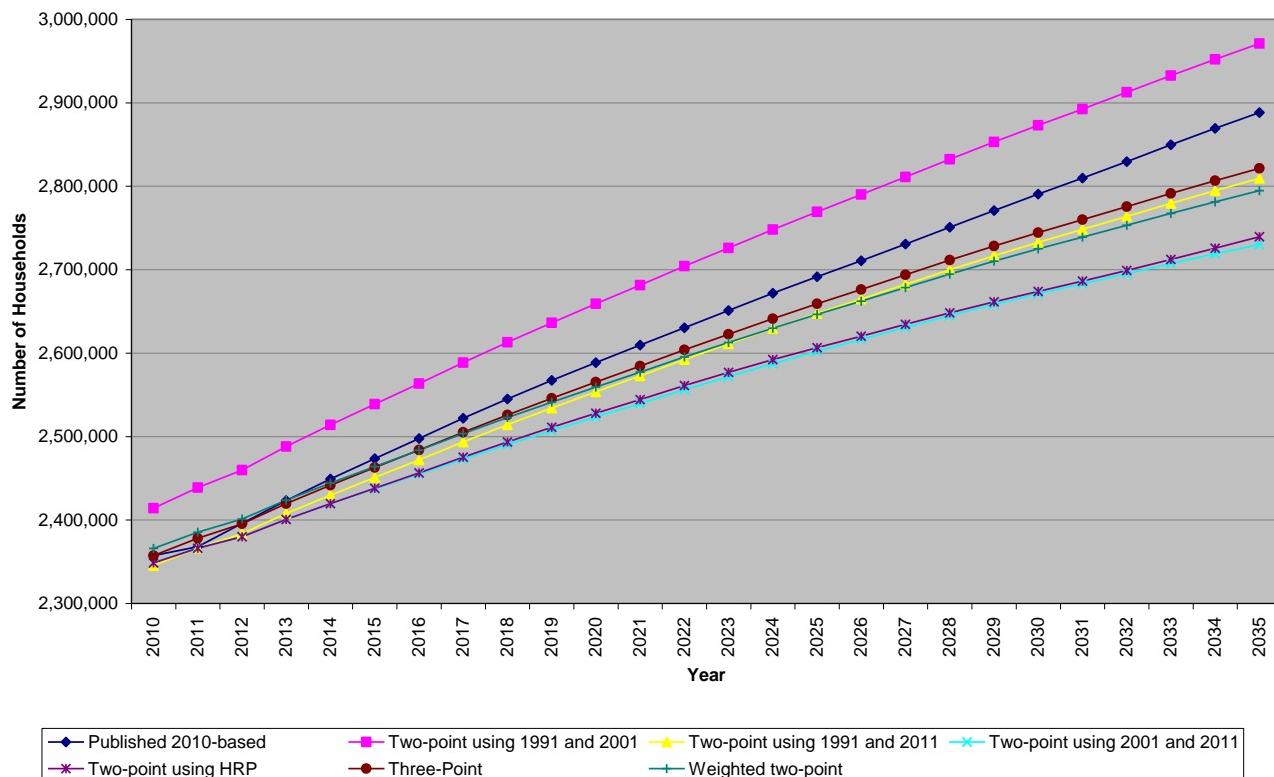
4.1 Total number of households projected

Scotland

[Figure 3](#) shows the total number of households throughout the projection period. The two-point exponential method using 1991 and 2001 Census data is by far the highest of the projections, this is not surprising as the trend projected is from a period of higher household growth. [Figure 3](#) also highlights the effect of constraining the projection to the household estimates, with the published figures being considerably lower than unconstrained figures.

The two point exponential method using the 2001 and 2011 Censuses is noticeably lower than the other sets of projections, whether using the head of household or household reference person. This is partly due to the fact that trends projected by this method are affected the most by the slowdown in household growth.

The other methods are all quite similar in terms of the projected total number of households. In 2035 the projected number of households for these methods range from 2.79 million households using the weighted two-point method to 2.82 million households using the three-point; a difference of less than 27,000 (one per cent).

Figure 3: Total number of households, 2010-2035, Scotland

When comparing the projections to the 2012 household estimates, [Table 1](#), we see that the two-point exponential method using the 1991 and 2001 Censuses has the largest difference at over three per cent above the household estimate. Again, this is not surprising given that this is the only method that does not incorporate headship rates from the 2011 Census.

The two-point exponential method using the 1991 and 2011 Censuses was lower than the household estimate by just over 2,500 households (-0.1 per cent), this was the closest any of method got to matching the households estimates.

However all of the methods were well within one per cent of the household estimates, with the weighted two-point method doing worst being nearly 15,000 households (0.6 per cent) higher than the household estimate. With a different choice of weights, the difference between the weighted two-point method and the household estimates could be reduced, however this increases the differences in the distribution of household types when compared to the SHS data.

Again when looking at the two-point projection using the 2001 and 2011 Censuses there isn't much difference between using the head of household or the household reference person.

Note that with the exceptions of the two-point 1991 and 2001 projection, the 'order' of the projections changes over time. For example, the weighted two-point projection has the highest results in 2012 ([Table 1](#)), but in the later projection years, it is in the middle of the other projections.

Table 1: Comparison of the projections and the 2012 Household Estimates, Scotland

Projection method	Household Estimates	Published 2010-based	Two-Point 1991 & 2001	Two-Point 1991 & 2011	Two-Point 2001 & 2011	Two-point using HRP	Three-Point	Weighted Two-Point
Total number of households in 2012	2,386,207	2,395,899	2,459,676	2,383,611	2,380,188	2,379,809	2,395,348	2,401,014
Difference from 2012 household estimates	0	9,692	73,469	-2,596	-6,019	-6,398	9,141	14,807
Percentage difference from the 2012 household projections	0.0%	0.4%	3.1%	-0.1%	-0.3%	-0.3%	0.4%	0.6%

Council Areas

The projections were also compared to the 2012 Household Estimates at Council area level. Table 2 shows the percentage difference between the projections and the household estimates for each council area, as a percentage of the household estimates.

The performance of each method at Council area level largely follows the performance for Scotland overall. Again the two two-point exponential methods that include the 2011 Census perform best, both having the projected number of households within one per cent of the household estimates for 29 of the 32 Council areas. The two-point exponential method using 1991 and 2011 Censuses still has slightly better agreement with the household estimates, with 19 councils being within 0.5 per cent compared to 17 councils when projecting using the 2001 and 2011 Censuses. Again we see that there is very little difference between using the head of household or household reference person when using the two-point exponential method using the 2001 and 2011 Censuses.

The three-point exponential method actually results in the most councils with a percentage difference of under 0.5 per cent (20 councils), however the number of councils with a difference of greater than one per cent is also greater than when using the two-point exponential method.

Again the weighted two-point method has the largest differences among the methods using the 2011 Census.

Table 2: Percentage difference between the projections and the 2012 Household Estimates for Council Areas

Percentage differences from 2012 Household Estimates

Key:

- The percentage difference between the projection and the household estimates is greater than 1%
- The percentage difference between the projection and the household estimates is between 0.5% and 1%
- The percentage difference between the projection and the household estimates is less than 0.5%

Council Area	Published 2010-based	Two-point using 1991 and 2001	Two-point using 1991 and 2011	Two-point using 2001 and 2011	Two-point using HRP	Three-Point	Weighted two-point
Aberdeen City	1.9%	6.6%	0.8%	0.5%	0.5%	1.7%	2.1%
Aberdeenshire	0.2%	2.1%	-0.3%	-0.4%	-0.5%	0.0%	0.2%
Angus	-1.6%	3.0%	-0.2%	-0.3%	-0.3%	0.3%	0.5%
Argyll & Bute	4.0%	-1.3%	-2.0%	-2.0%	-1.9%	-1.9%	-1.8%
Clackmannanshire	2.0%	4.9%	-0.5%	-0.7%	-0.7%	0.3%	0.7%
Dumfries & Galloway	1.0%	1.3%	-0.6%	-0.6%	-0.6%	-0.3%	-0.1%
Dundee City	2.8%	1.5%	-0.1%	-0.1%	-0.1%	0.2%	0.3%
East Ayrshire	0.1%	0.2%	-0.6%	-0.7%	-0.7%	-0.5%	-0.5%
East Dunbartonshire	-1.0%	2.2%	0.0%	0.0%	0.0%	0.3%	0.6%
East Lothian	0.9%	2.2%	-0.2%	-0.4%	-0.4%	0.1%	0.3%
East Renfrewshire	-3.3%	-0.9%	-0.6%	-0.6%	-0.6%	-0.7%	-0.7%
Edinburgh, City of	1.0%	3.7%	0.8%	0.7%	0.7%	1.2%	1.5%
Eilean Siar	-3.8%	0.9%	-1.2%	-1.3%	-1.3%	-0.9%	-0.7%
Falkirk	0.7%	4.6%	0.0%	-0.2%	-0.2%	0.7%	1.0%
Fife	1.2%	4.1%	-0.2%	-0.4%	-0.4%	0.5%	0.8%
Glasgow City	0.3%	6.5%	0.5%	0.2%	0.1%	1.5%	1.8%
Highland	0.7%	2.9%	-0.6%	-0.7%	-0.8%	-0.1%	0.2%
Inverclyde	-2.4%	2.4%	-0.4%	-0.5%	-0.5%	0.0%	0.2%
Midlothian	1.4%	2.2%	-0.6%	-0.8%	-0.8%	-0.2%	0.0%
Moray	-1.5%	-0.7%	-1.5%	-1.5%	-1.5%	-1.4%	-1.3%
North Ayrshire	-0.3%	2.4%	-0.3%	-0.5%	-0.4%	0.1%	0.3%
North Lanarkshire	-0.1%	2.5%	-0.6%	-0.7%	-0.7%	-0.1%	0.1%
Orkney Islands	-3.4%	3.6%	0.2%	0.1%	0.1%	0.7%	1.0%
Perth & Kinross	1.7%	4.1%	0.4%	0.3%	0.3%	1.0%	1.3%
Renfrewshire	-1.7%	0.4%	-0.3%	-0.3%	-0.3%	-0.2%	-0.2%
Scottish Borders	0.1%	1.7%	-0.4%	-0.5%	-0.4%	-0.1%	0.1%
Shetland Islands	-0.2%	4.9%	-0.7%	-0.9%	-1.0%	0.1%	0.6%
South Ayrshire	1.0%	1.4%	-0.2%	-0.3%	-0.3%	0.0%	0.1%
South Lanarkshire	-0.6%	1.7%	-0.4%	-0.5%	-0.5%	-0.1%	0.1%
Stirling	2.6%	3.4%	0.0%	-0.2%	-0.2%	0.5%	0.8%
West Dunbartonshire	-1.0%	1.3%	-0.2%	-0.3%	-0.3%	0.0%	0.1%
West Lothian	1.0%	3.4%	0.0%	-0.1%	-0.1%	0.5%	0.8%
Scotland	0.4%	3.1%	-0.1%	-0.3%	-0.3%	0.4%	0.6%

Number of council areas

Green (within 0.5%)	7	2	19	17	18	20	15
Orange (within 1.0%)	7	3	10	12	11	7	10
Red (outwith 1.0%)	18	27	3	3	3	5	7

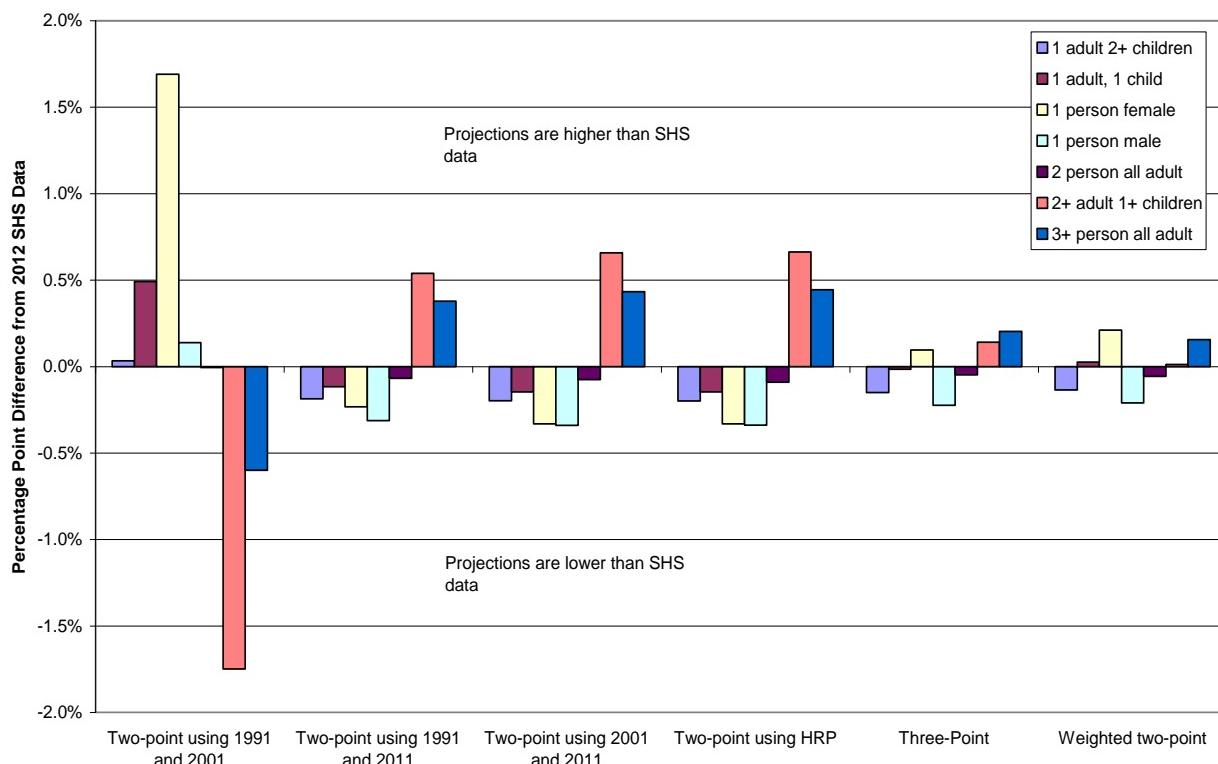
4.2 Household types, Scotland

Data from the 2012 Scottish Household Survey (SHS) was used to estimate the distribution of households by type in Scotland. The survey is designed to provide accurate, up-to-date information about the characteristics, attitudes and behaviour of Scottish households and individuals on a range of issues. This comparison was only made for the whole of Scotland as we felt the sample size of this survey would not provide reliable results for all council areas.

Table 3 provides a summary of the distribution of the household types for each projection and the 2012 SHS, while Figure 4 presents the percentage point difference between the projections and the SHS.

Table 3: Proportion of households by household type, 2012, Scotland

Household Type	Projection						Weighted two-point
	2012 Scottish Household Survey	Two-point using 1991 and 2001	Two-point using 1991 and 2011	Two-point using 2001 and 2011	Two-point using HRP	Three-Point	
1 adult 2+ children	2.9%	2.9%	2.7%	2.7%	2.7%	2.8%	2.8%
1 adult, 1 child	3.8%	4.2%	3.6%	3.6%	3.6%	3.7%	3.8%
1 person female	18.9%	20.6%	18.6%	18.5%	18.5%	19.0%	19.1%
1 person male	16.6%	16.8%	16.3%	16.3%	16.3%	16.4%	16.4%
2 person all adult	30.6%	30.6%	30.6%	30.6%	30.6%	30.6%	30.6%
2+ adult 1+ children	18.8%	17.0%	19.3%	19.5%	19.5%	18.9%	18.8%
3+ person all adult	8.4%	7.8%	8.8%	8.8%	8.9%	8.6%	8.6%

Figure 4: Percentage point difference between projections and the 2012 SHS for the percentage of households by household type

In this case the three-point exponential and weighted two-point methods perform best, with the weighted two-point method being marginally closer to the SHS distribution. For both of these methods all household types are within 0.25 percentage points of the 2012 SHS data.

The two-point exponential methods that incorporate the 2011 Census have similar distributions of household types, with the larger households being over-estimated and smaller households being under-estimated when compared to the SHS. In both cases, figures for 2+ adult households with children are over 0.5 percentage points greater than in the SHS.

Figures 5a and 5b show the projected number of households by type, for each projection method, for 2012 and 2035. Looking at the projected number of households by the type of household in 2035, Figure 5b, we see that the two-point exponential method using the 2001 and 2011 Censuses gives the lowest projection for smaller households and the highest for the larger households. The opposite pattern is seen in the two-point exponential using the 1991 and 2001 Censuses as this method produces the highest projection of smaller household and the highest projection of large households.

The two-point exponential method using the 1991 and 2011 Censuses and the three-point exponential method produce very similar numbers of households for each type, with the largest difference being found in one person female households where the three-point method is greater by just over 10,000 households. The weighted two-point method also has a similar distribution of household types to these models, however using this method results in a slightly lower number of smaller households and a higher number of large households.

Figure 5a: Projected number of households, by type, Scotland, 2012

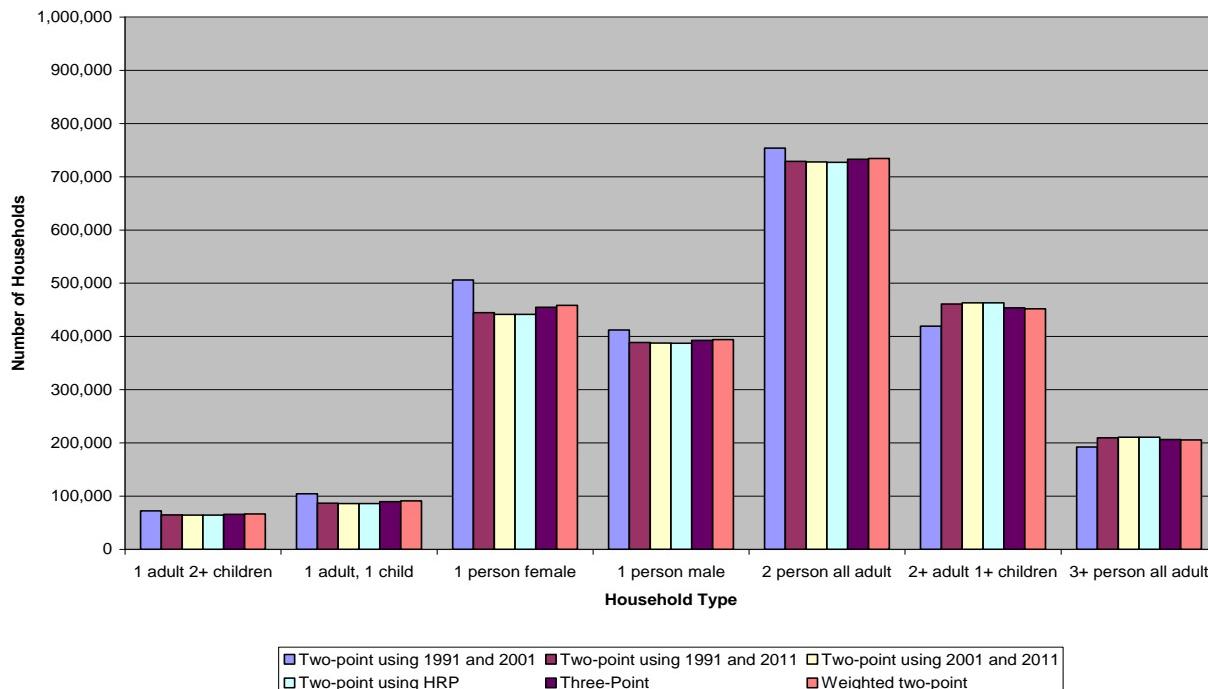
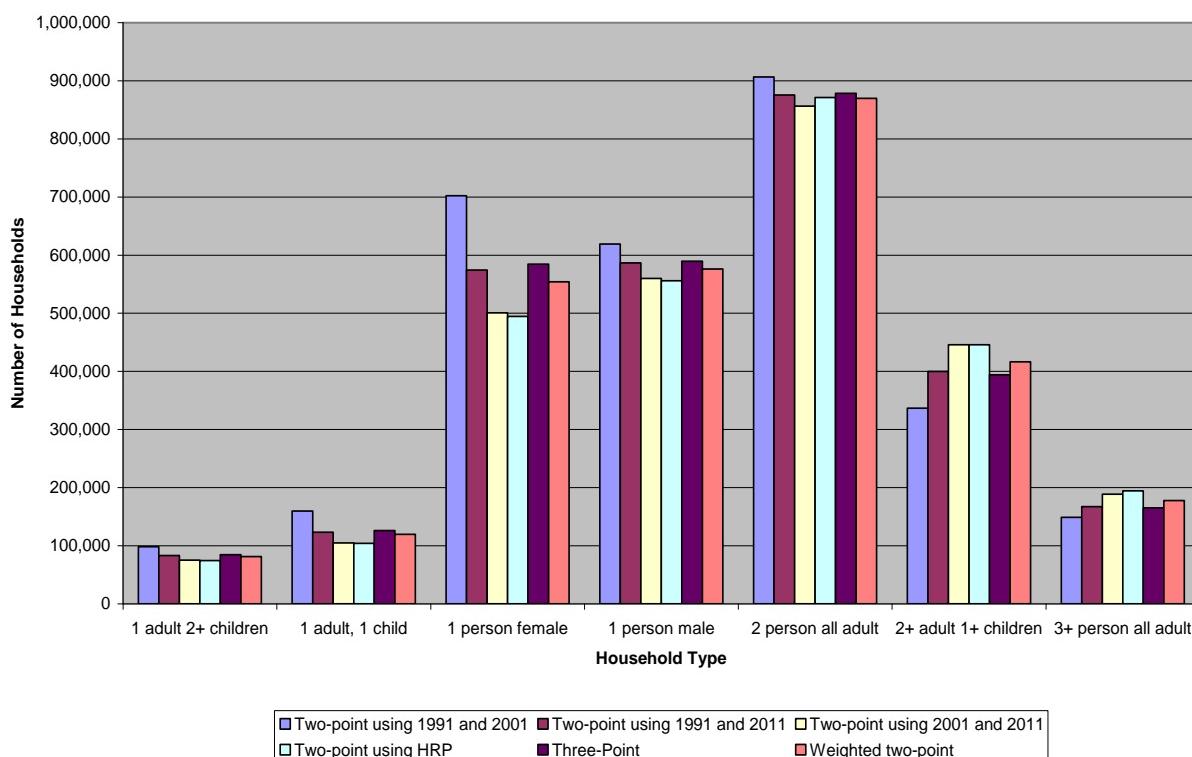


Figure 5b: Projected number of households, by type, Scotland, 2035



4.3 Age of head of household by household type, 2012-2035

Figures 6a to 10b show the age distributions for various household types in 2012 and 2035.

For households with one adult, with or without children, the two-point exponential method using the 2001 and 2011 Censuses and the two-point exponential method using the household reference person are virtually identical in 2012 and 2035, as the head of household and household reference person are nearly always the same person in these household types. For one person households we see that in 2035 all of the projections incorporating the 2011 Census project lower numbers of households in all age groups except from 55-69 when compared to the two-point exponential using the 1991 and 2001 Censuses.

For the other household types all of the projections are pretty similar in 2012, however there are some considerable differences by 2035. For example, the two-point exponential method using the household reference person has slightly different peaks for some household types such as two adults with no children.

Generally the projections that produce the most ‘extreme’ values are the two-point exponential using the 1991 and 2001 Censuses and the two-point exponential using the 2001 and 2011 Censuses (using either head of household or household reference person). The projections produced using either two-point exponential method using the 1991 and 2011 Censuses, the three-point exponential method or the weighted two-point exponential method are all quite similar for all household types.

Figure 6a: Number of households by the age of the head of household for one person households in 2012, Scotland

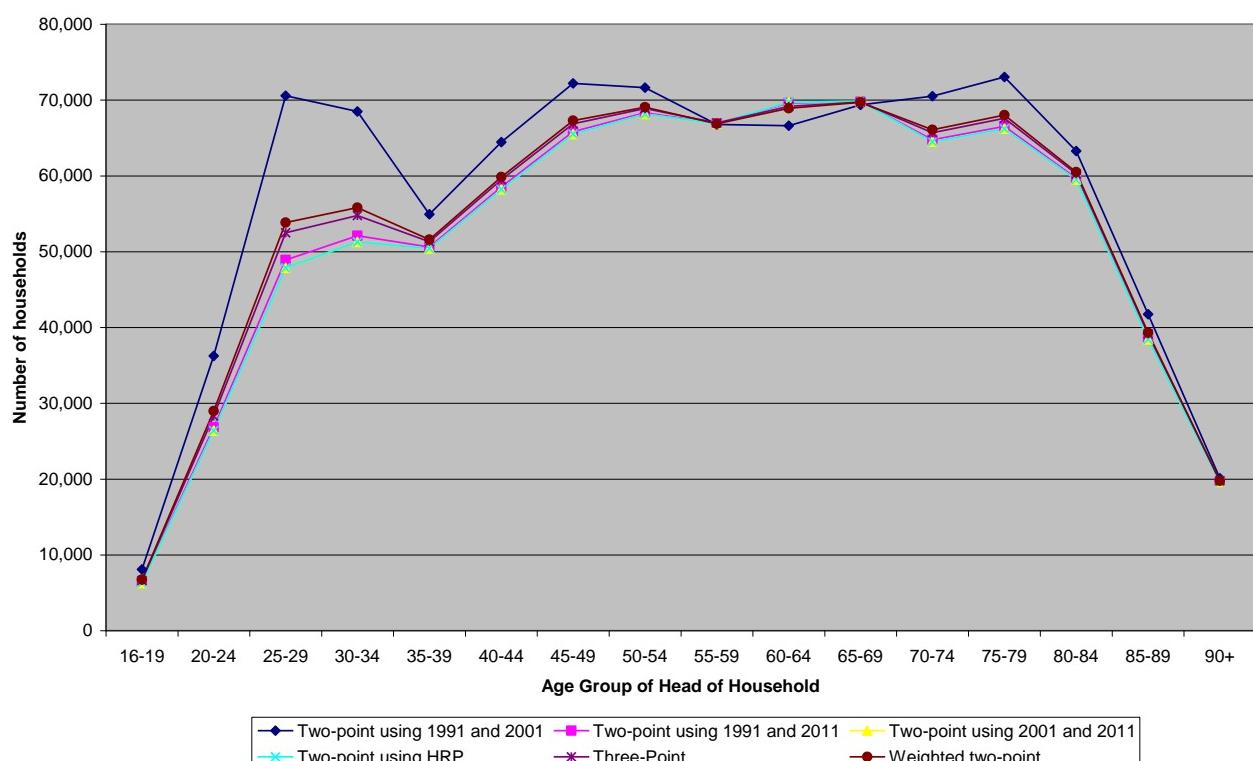


Figure 6b: Number of households by the age of the head of household for one person households in 2035, Scotland

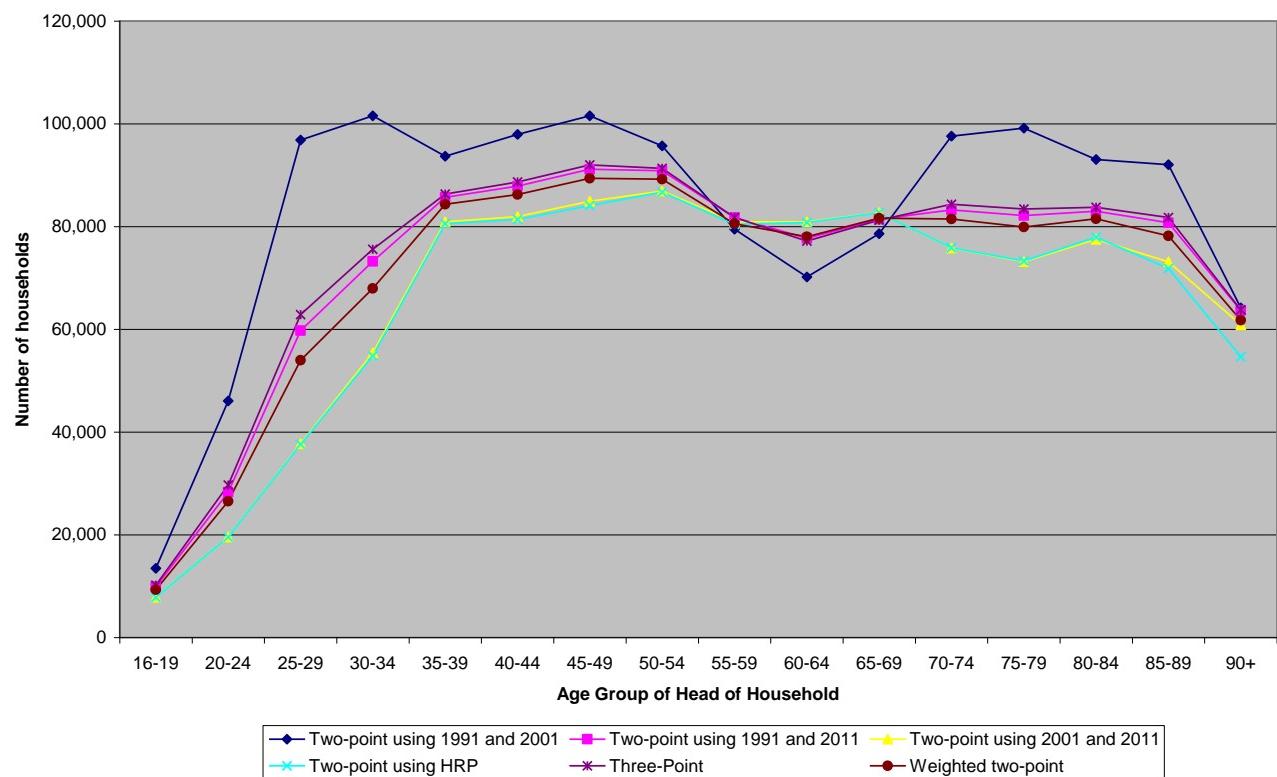


Figure 7a: Number of households by the age of the head of household for households with one adult and one or more children in 2012, Scotland

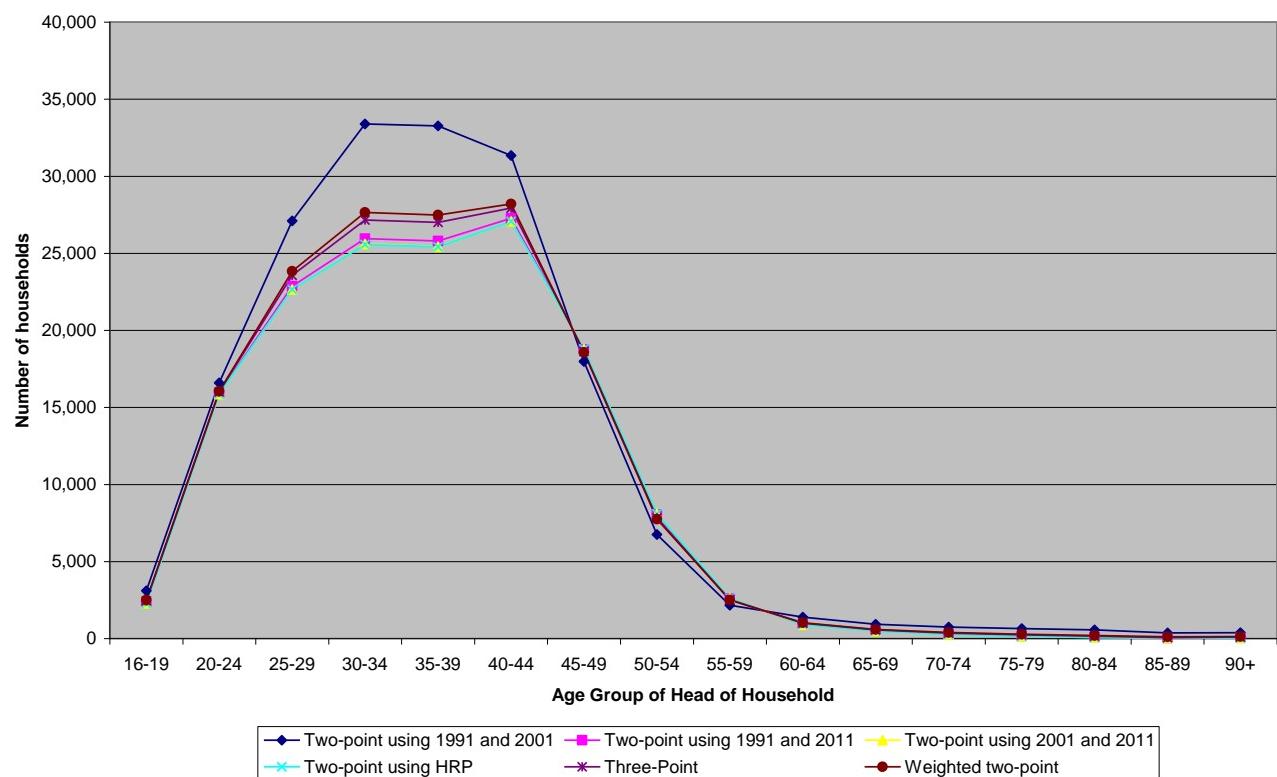


Figure 7b: Number of households by the age of the head of household for households with one adult and one or more children in 2035, Scotland

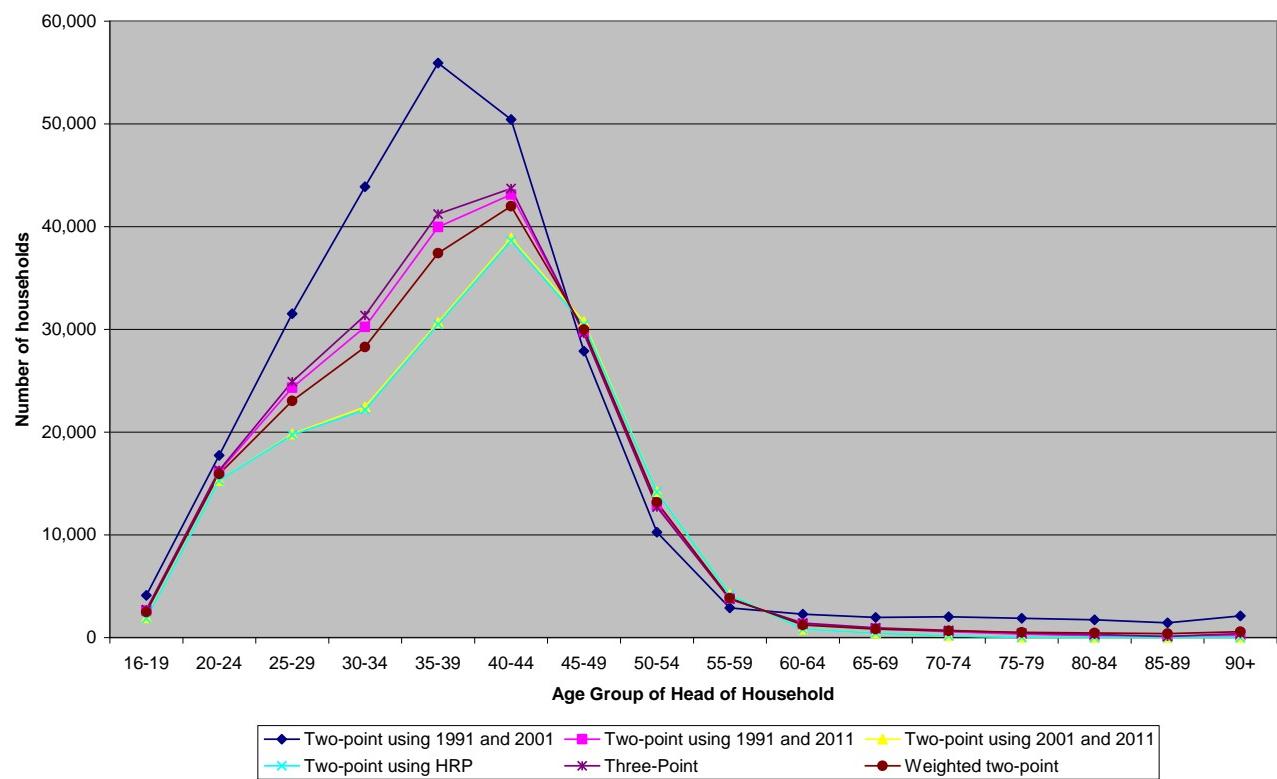


Figure 8a: Number of households by the age of the head of household for households with two adults and no children in 2012, Scotland

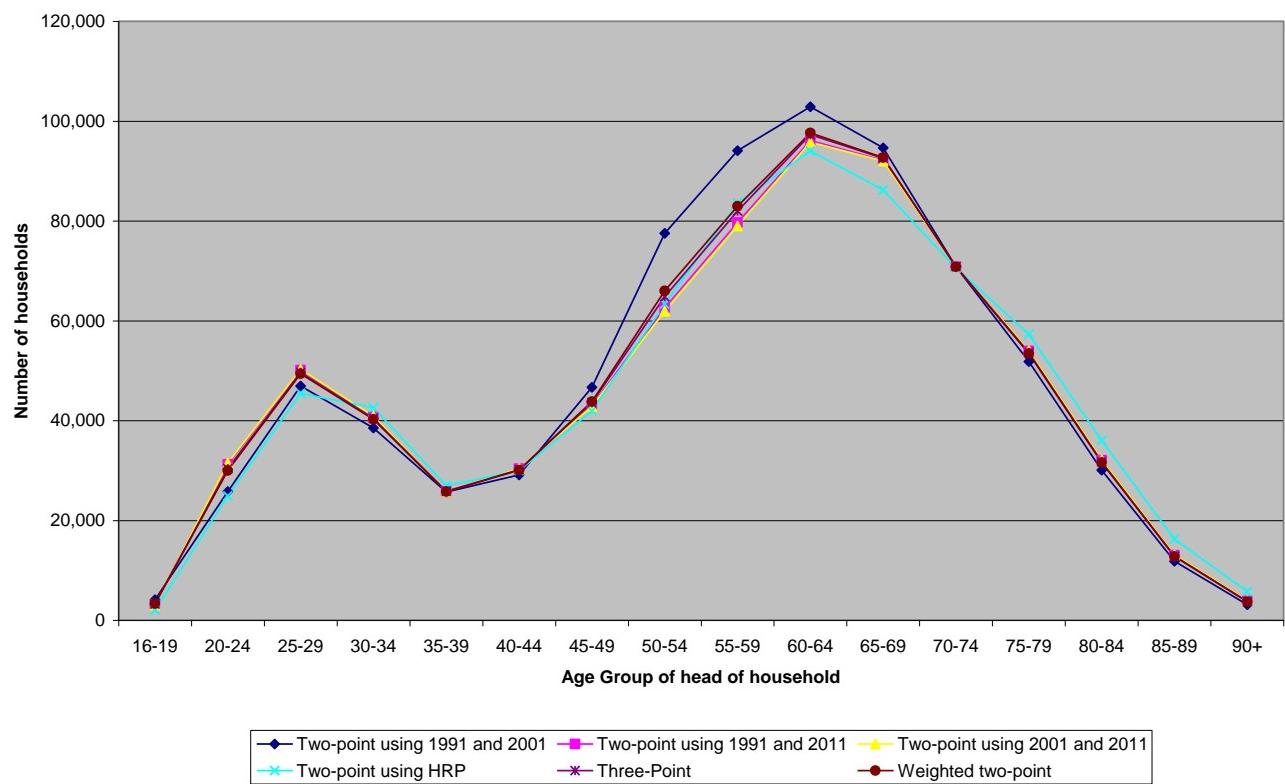


Figure 8b: Number of households by the age of the head of household for households with two adults and no children in 2035, Scotland

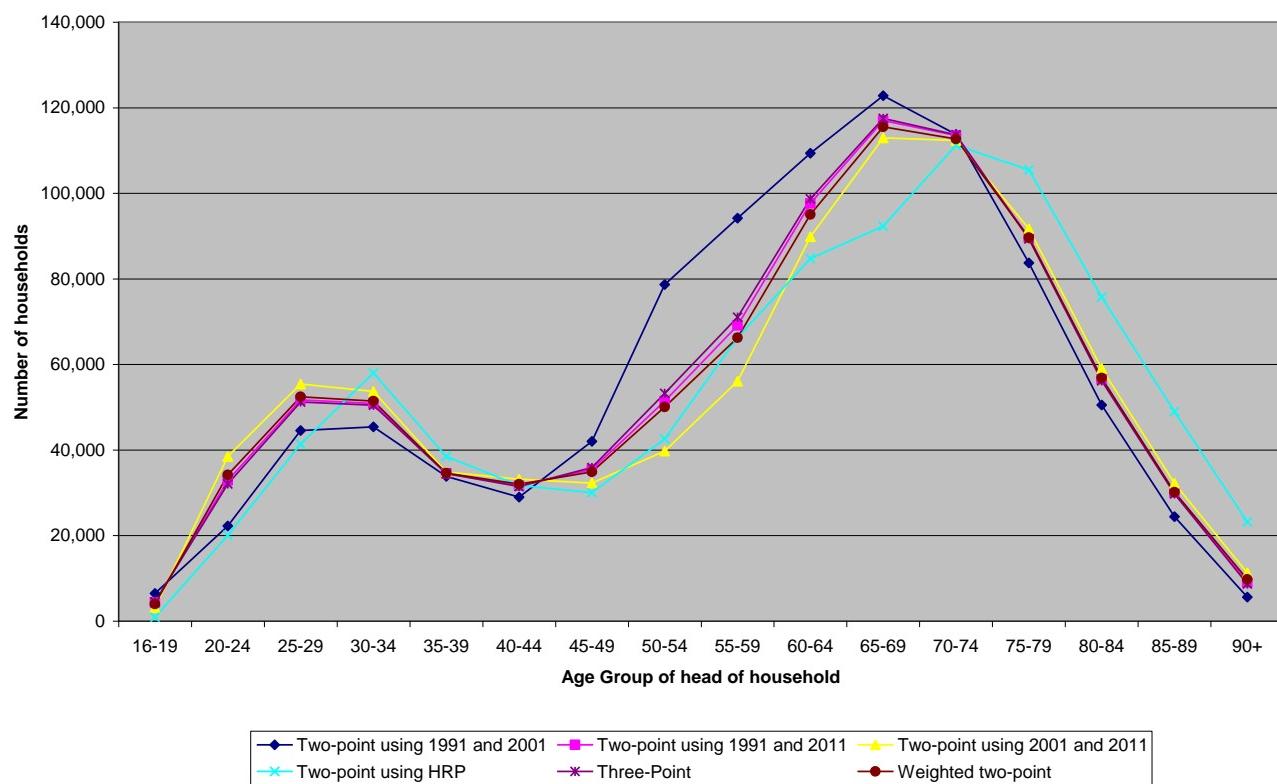


Figure 9a: Number of households by the age of the head of household for households with two or more adults and one or more children in 2012, Scotland

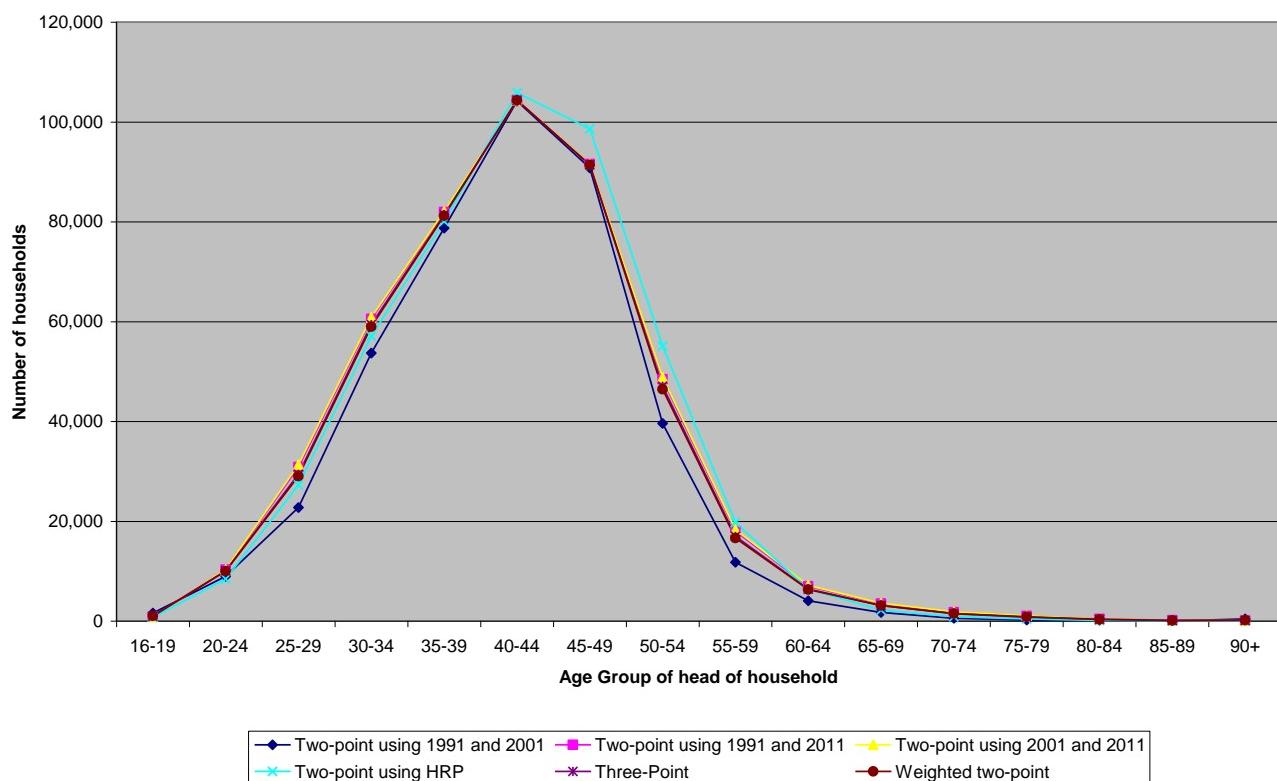


Figure 9b: Number of households by the age of the head of household for households with two or more adults and one or more children in 2035, Scotland

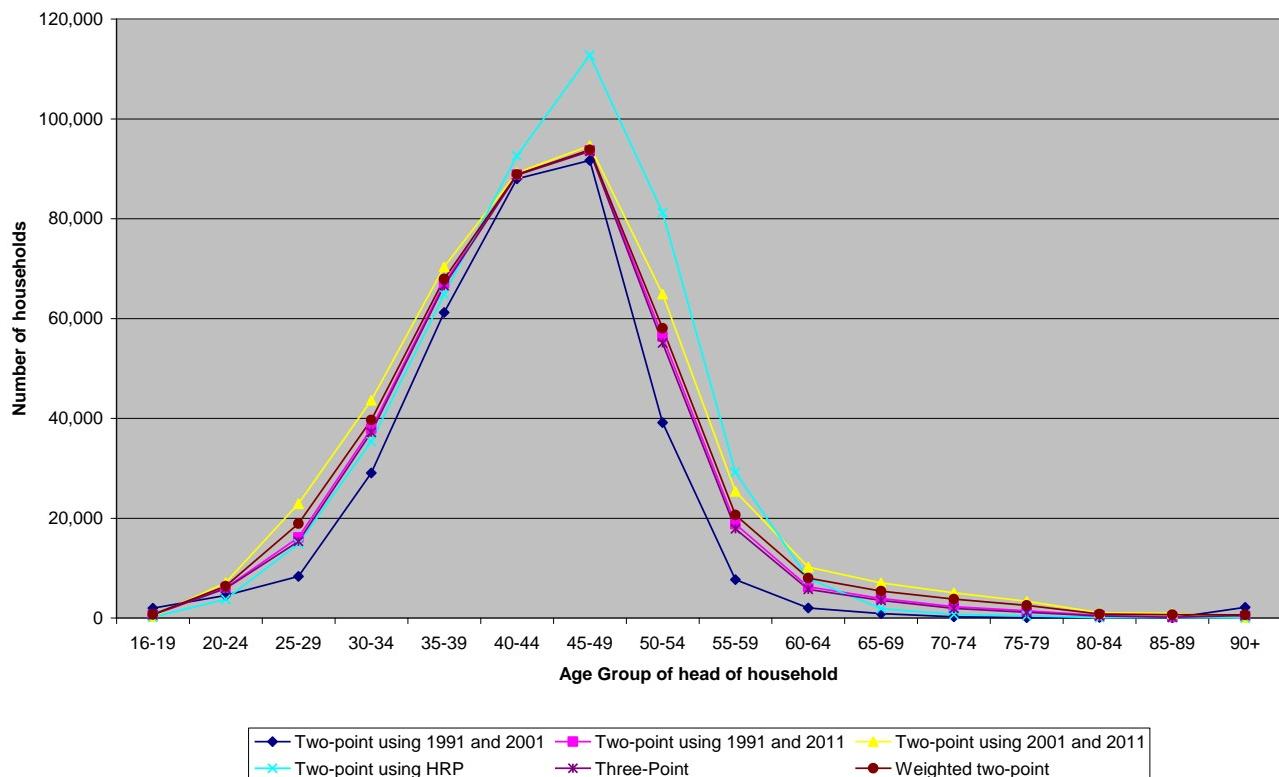


Figure 10a: Number of households by the age of the head of household for households with three or more adults and no children in 2012, Scotland

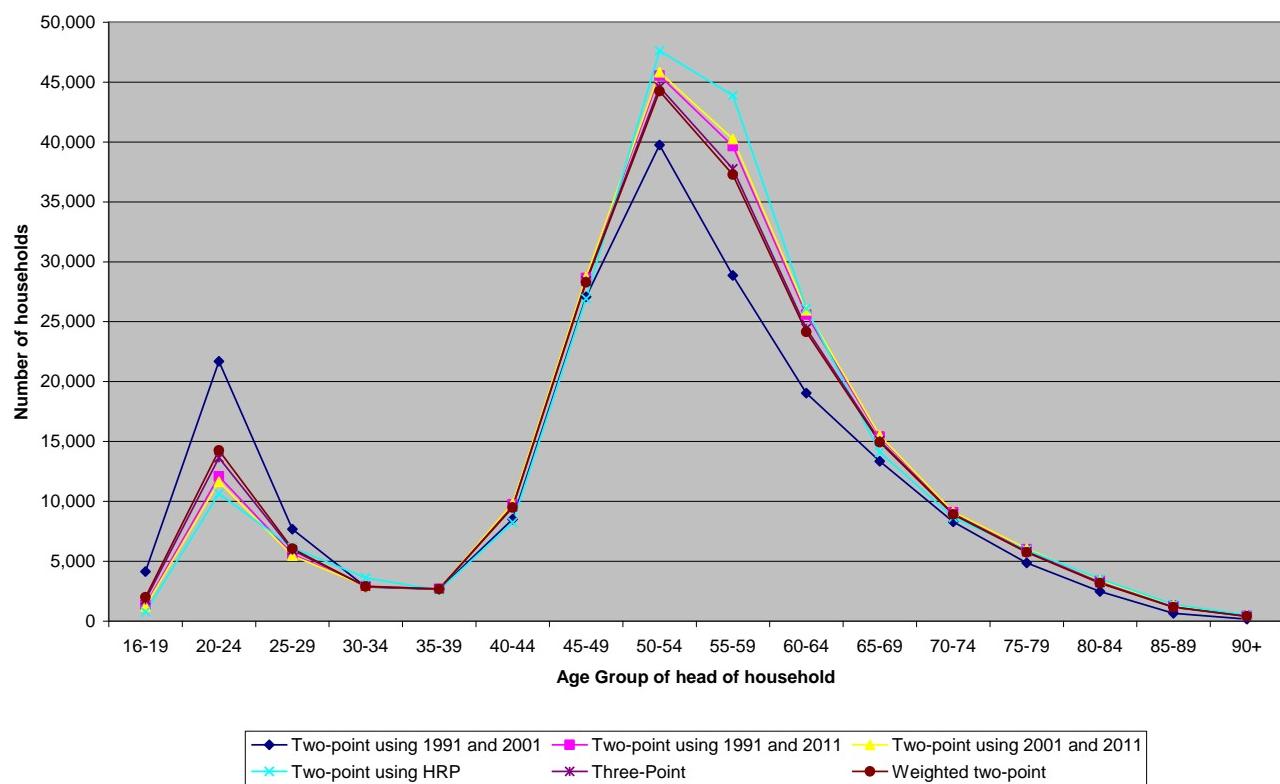
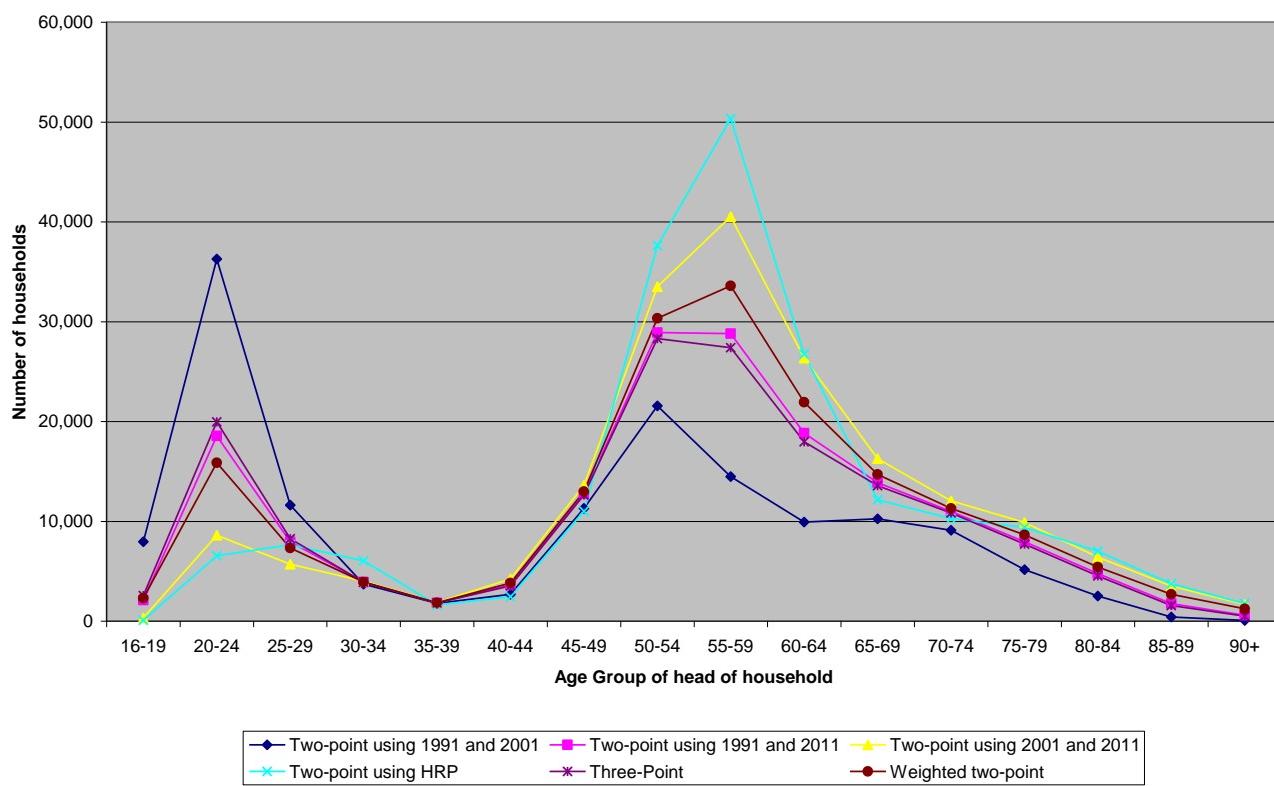


Figure 10b: Number of households by the age of the head of household for households with three or more adults and no children in 2035, Scotland



5. Results from full process

This section compares some of the results from the household projections after they have been constrained and any minimum adult/child adjustments have been made. It is a good sign if the constraining makes little difference, and if few adjustments are required to the projections.

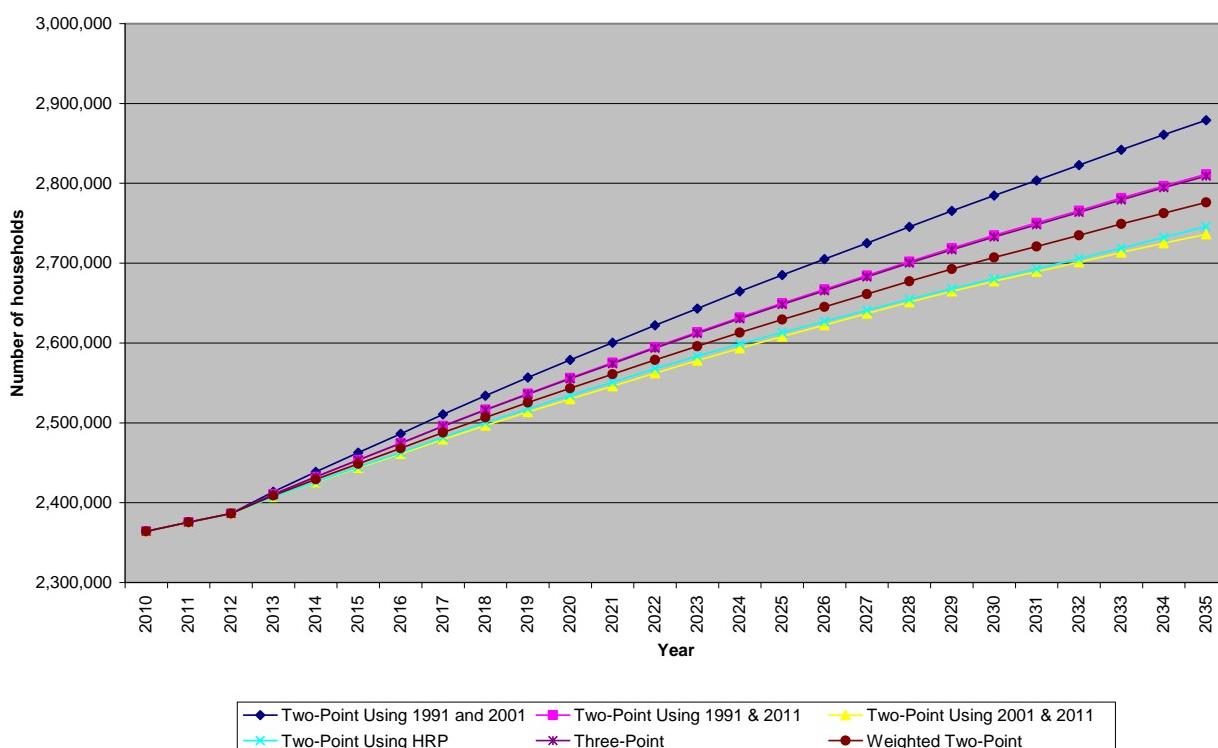
The method used to make minimum adult/child adjustments has been changed slightly. Previously, if a Council area needed an adjustment for any year in the projection period then, for that council, every year in the projection period received the same adjustment. This has been changed so that in a council where minimum adult/child adjustments are necessary, adjustments are only performed on the years where this is necessary and each adjustment is calculated individually to minimise the adjustment that is made.

We would welcome comments from Household Analysis Review Group (HARG) members regarding the changes that have been made to method for minimum adult/child adjustments.

None of the methods required minimum child adjustments, while only the two-point exponential method using the 1991 and 2001 Censuses required minimum adult adjustments (adjustments for Edinburgh in 2033-35 and Eilean Siar in 2035).

Figure 11 shows the projected total number of households in Scotland after the constraining process. This is very similar to Figure 3 but the projections have been shifted slightly higher or lower depending on the difference there was with the 2012 household estimates. This means there is a larger difference between the weighted two-point method, which produces a projection that is more noticeably lower than the three-point method or the two-point method using the 1991 and 2011 Censuses. It also means that the three-point method and the two-point exponential method using 1991 and 2011 are very similar, differing by just over 2,250 households in 2035.

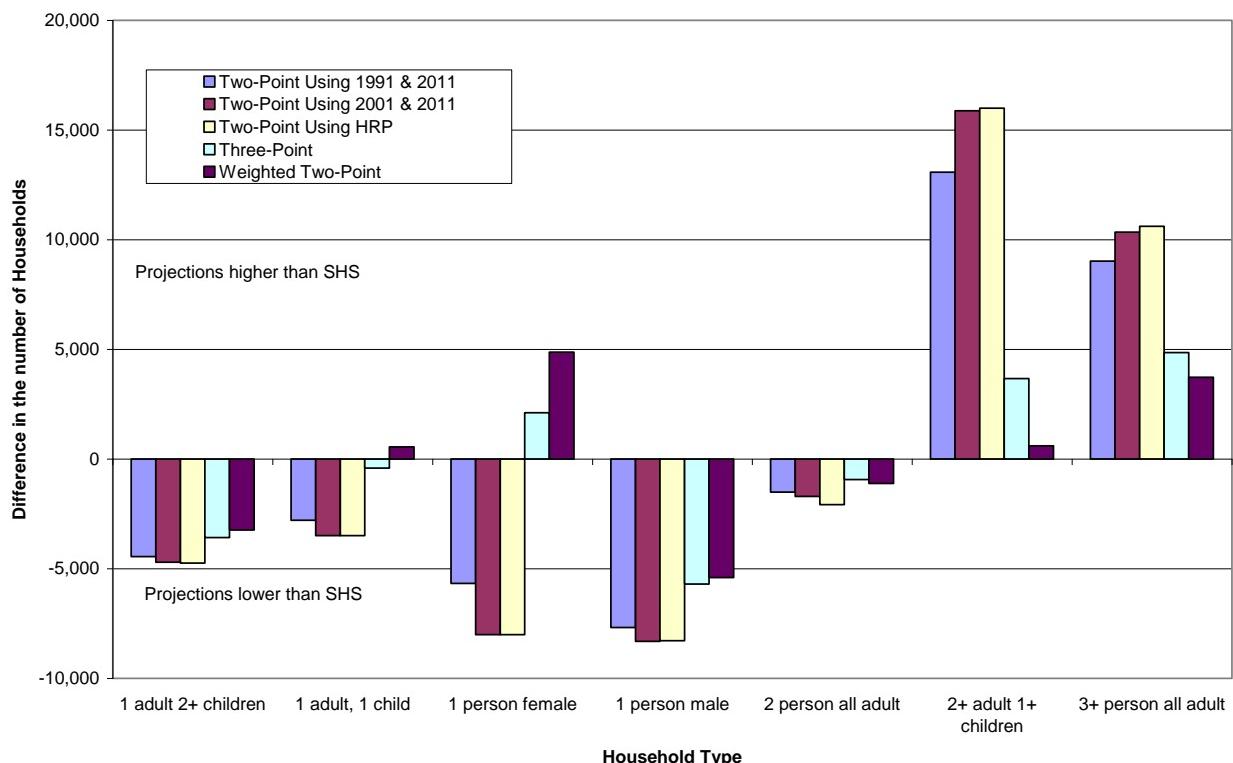
Figure 11: Total number of households, 2010-2035, Scotland. Projections constrained to 2010, 2011 and 2012 household estimates.



When comparing the distribution of households by type in 2012 to the 2012 Scottish Household Survey (SHS) data, the best agreement is still found when using the weighted two-point method. However the three-point method still performs very well in this respect as well.

Both of these methods perform considerably better in this comparison than any of two-point exponential methods.

Figure 12: Difference from the number of households in the 2012 SHS, by type



6. Conclusion

Based on the testing that we have done, it appears that the models that perform best are the two-point exponential using the 1991 and 2011 Censuses, the three-point exponential method and the weighted two-point exponential method.

While the two-point exponential method using the 1991 and 2011 Censuses performs well, particularly in the total number of households, the other two methods produce a distribution of household types that is much closer to the 2012 SHS data. Another problem with this method is that there isn't a particularly good reason to ignore the 2001 Census.

The three-point exponential method and the weighted two-point method both project the household types very well, with the weighted method being fractionally better. However the three-point method produces a projected total number of households that is closer to the household estimates, minimising the size of the adjustments that are made. The main advantage that the weighted method has is that for future projections, it would allow us to take recent changes in household composition into account by adjusting the weights as new data becomes available.

Conclusion: We propose using the weighted two-point exponential model for the household projections due to the flexibility that this method has over the others. This involves the use of headship rates from the 1991, 2001 and 2011 Censuses, based on the head of household as defined in previous years. We would welcome the views of HARG members on this, and on whether there is anything else we should consider.

NRS: Household estimates and projections branch

December 2013

Annex A – Mathematics of the different methods of projection

Two-point exponential method

$$y_i = k + ab^{x_i}$$

where i = the year in the projection period

y_i = the headship rate in year i

$$\begin{aligned} k &= 1 \quad \text{if } y_{2011} \geq y_{2001} \\ \text{or } k &= 0 \quad y_{2011} < y_{2001} \end{aligned}$$

$$\begin{aligned} a &= y_{2001} - k \\ b &= \frac{y_{2011} - k}{y_{2001} - k} \\ x_i &= \frac{i - 2001}{2011 - 2001} \end{aligned}$$

Three-point exponential method

The formula for the three-point exponential method is as follows:

$$\begin{aligned} y_i &= \exp(\hat{z}^*(2001 - i) - \bar{z}) \quad \text{if } x < 0 \\ \text{or } y_i &= 1 - \exp(\hat{z}^*(2001 - i) - \bar{z}) \quad x \geq 0 \end{aligned}$$

Where $c = 1991, 2001$ or 2011

i = the year in the projection period

y_i = headship rate in year i

$$x = \frac{\sum_c ((c - 2001) * (y_c - \frac{c}{3}))}{200}$$

$$\hat{z} = \frac{\sum_c ((c - 2001) * (z_c - \frac{c}{3}))}{200}$$

$$\begin{aligned} z_c &= -\log(y_c) \quad \text{if } x < 0 \\ \text{or } z_c &= -\log(1 - y_c) \quad x \geq 0 \end{aligned}$$

$$\bar{z} = \frac{\sum_c z_c}{3}$$